$Q = -k(\partial T/\partial y)_{y=0}$ is then

$$\left.\begin{aligned} Q &= 0.494k(T_w - T_\infty)\frac{S^{1/2}(X)R^n(X)}{\left(\int_0^X S^{1/2}(t)R^{2n}(t)\,dt\right)^{1/2}} \\ &\quad\times \frac{1}{l}\left(\frac{\nu l}{\chi\alpha}\right)^{1/2} Ra^{1/4} \\ &= 0.494\frac{k(T_w - T_\infty)}{l}\left(\frac{l}{x}\right)^{1/2} Ra^{1/4}\left(\frac{\nu l}{\chi\alpha}\right)^{1/4} \end{aligned}\right\} \quad (10)$$

for a semi-infinite, vertical plate which is identical to the results as obtained in ref. [4]. For a cylinder we set $n = 0$ and $S(X) = \sin X$ whilst for a sphere $n = 1$ and $S(X) = \sin X$.

It is observed from equation (10) that $Q \propto Ra^{1/4}$ whereas for a Darcian fluid $Q \propto Ra^{1/2}$. Fand *et al.* [3] found, experimentally, that for a cylinder the Rayleigh number dependence on the local heat transfer varied from $Ra^{0.694}$ at low Rayleigh numbers to $Ra^{0.372}$ for higher Rayleigh numbers (but still less than about 200). The results presented in this note confirm the conclusions made in ref. [3] that at high Rayleigh numbers non-Darcian effects are very important.

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A further examination of void fraction in annular two-phase flow

J. J. J. Chen

Chemical and Materials Engineering Department, The University of Auckland, Private Bag, Auckland, New Zealand

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INTRODUCTION AND LITERATURE

A SET of semi-empirical equations for the prediction of void fraction in annular gas–liquid flow had been derived by Tandon *et al.* [1]. These equations are rather cumbersome and consist of three separate equations to cover the entire range of flow rates. This is due to the fact that the derivation had been based on the semi-empirical fragmental representation of velocity distribution due to Von Karman and the Soliman *et al.* [2] curve-fit of the Lockhart–Martinelli data [3].

Butterworth [4] had shown that a number of the more commonly used holdup prediction equations may be represented by the following relationship:

$$\left[\frac{1-\alpha}{\alpha}\right] = A\left[\frac{1-x}{x}\right]^p\left[\frac{\rho_G}{\rho_L}\right]^q\left[\frac{\mu_L}{\mu_G}\right]^r. \quad (1)$$

The homogeneous model, the correlations due to Zivi [5], Turner and Wallis [6], Lockhart and Martinelli [3], Thom [7] and Baroczy [8] may all be shown to be expressible in the form of equation (1).

Chen and Spedding [9] also analysed the idealised annular flow situation and obtained:

$$\alpha = \frac{1}{1+X^{2/3}}. \quad (2)$$

For the case when both the gas and the liquid are flowing in the turbulent regime, X is given by:

$$X_{tt} = (\mu_L/\mu_G)^{0.1}(\rho_G/\rho_L)^{0.5}\left[\frac{1-x}{x}\right]^{0.9}. \quad (3)$$

It was found that equation (2) did not represent well the data available and consequently, an empirical parameter, k, was introduced to result in:

$$\alpha = \frac{k}{k+X^{2/3}}. \quad (4)$$

It is of interest to note at this point that Chen and Spedding [10, 11] analysed the form of equation (1) as given by Butterworth [4] and found that this form of equation may in fact be derived for the case of ideal stratified and ideal annu-

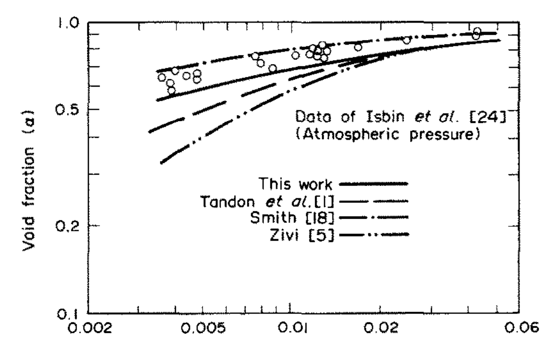

FIG. 1. Comparison of void fraction correlations with experimental data of ref. [24] for the steam–water system at atmospheric conditions.

NOMENCLATURE

A parameter in equation (1)
k parameter in equation (4)
p parameter in equation (1)
q parameter in equation (1)
r parameter in equation (1)
X Lockhart–Martinelli parameter
x vapour quality.

Greek symbols
α void fraction
ρ density [$kg\,m^{-3}$]
μ viscosity [$kg\,s^{-1}\,m^{-1}$].

Subscripts
G gas (vapour) phase
L liquid phase
tt liquid–gas turbulent–turbulent flow.

lar flows. Chen and Spedding [10, 11] further showed that in addition to the void fraction correlations considered by Butterworth [4], the voidage correlations of Armand [12], Nguyen and Spedding [13], Zuber and Findlay [14], Griffith and Wallis [15], Nicklin *et al.* [16], Bankoff [17], and the equal velocity head model of Smith [18] may all be cast in the same form as equation (1), although some of these had additional factors incorporated. It is appropriate at this point to note that Tandon *et al.* [1] had compared their results with those of Zivi [5] and Smith [18].

Returning to equation (4), it was postulated that k is a function of a number of parameters including the pipe size,

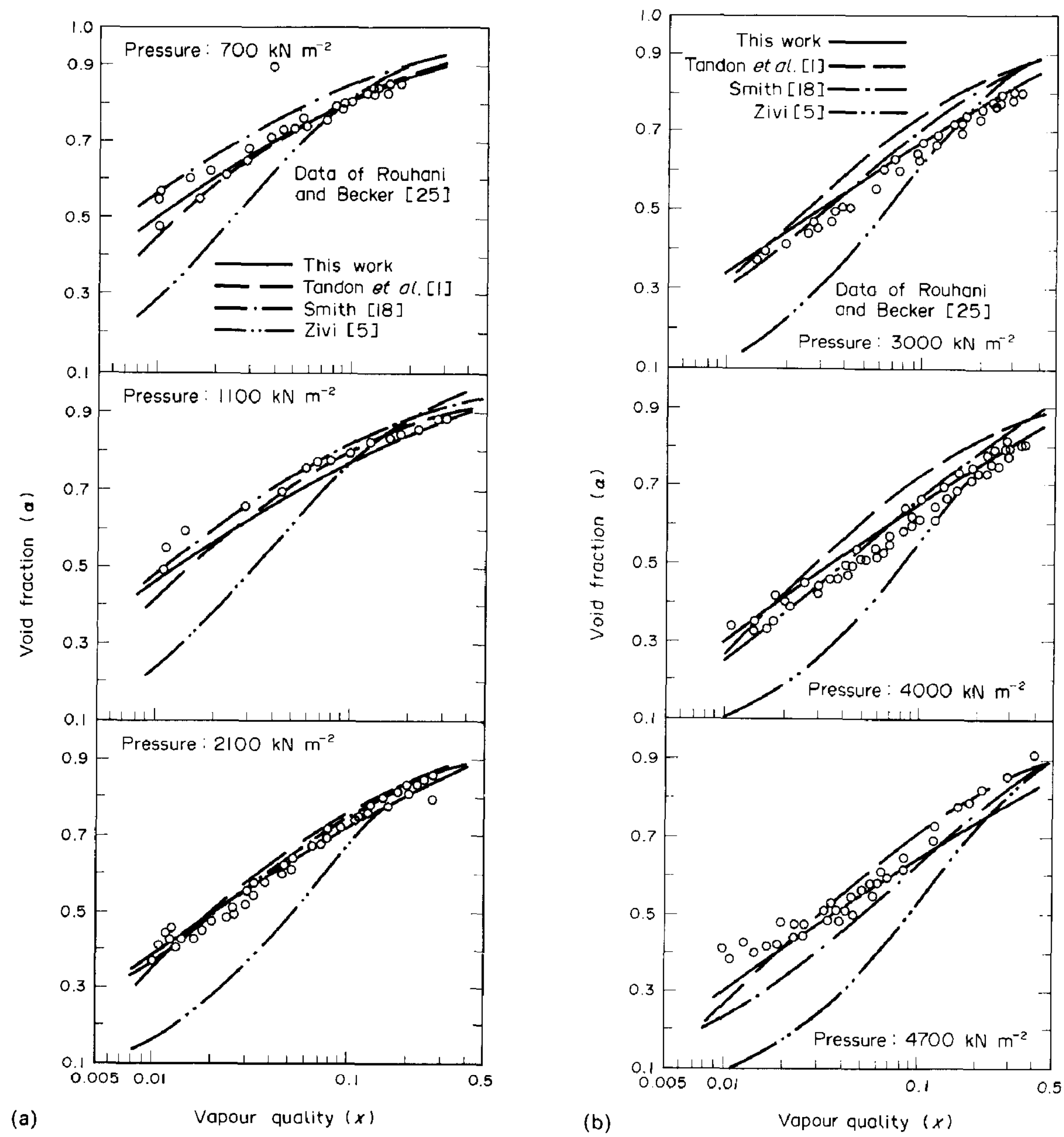

FIG. 2. (a)–(c) Comparison of void fraction correlations with experimental data of ref. [25] for the steam–water system at various pressures.

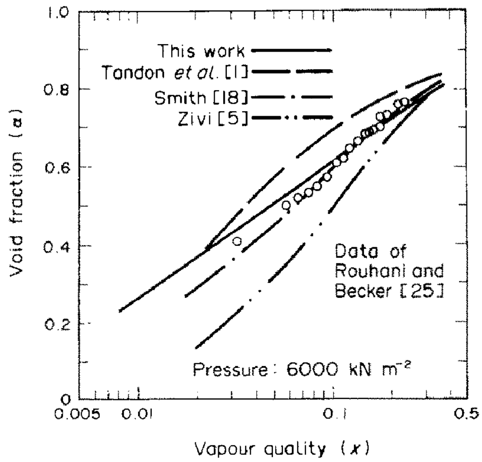

FIG. 2(c).

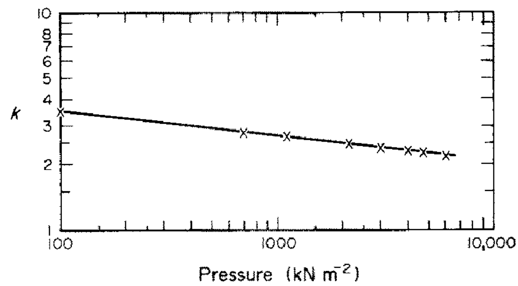

FIG. 3. Value of k as a function of system pressure used in equation (4) for the predictions shown in Figs. 1 and 2.

and factors which influence the liquid film such as the interfacial characteristics and the skewed velocity profile [9, 19]. It was also shown that with $k = 6.0$, equation (4) matches the experimental data obtained from the air–water system at atmospheric conditions flowing through a 0.0455-m pipe. With $k = 2.5$, equation (4) matches the geothermal steam–water flow data in the pressure range of 440–1210 kN m^{-2} from a 0.2-m pipe [9]. In addition, when $k = 3.5$, equation (4) matches the Lockhart–Martinelli data [9, 19]. Recently, Chen [20] showed that if k was taken as being a function of X, equation (4) may also be used to represent the void fraction data of Richardson *et al.* [21, 22] for the case of two-phase flow of air and china clay suspensions in horizontal and vertical pipe lines, as well as when the liquid was inelastic and viscoelastic. It should be noted that equation (4) may be readily expressed in the same form as equation (1), and that equation (1) had also been used to analyse a wide range of data including both inclined upward and downward flows [10, 11, 23].

ANALYSIS AND COMPARISON

Equation (4) has been shown to be valid for a wide range of conditions. It would therefore be useful to check if it is also valid for the data of Isbin *et al.* [24] and Rouhani and Becker [25] as presented by Tandon *et al.* [1]. Inspections of the predictions of Tandon *et al.* [1] show that they are reasonable at pressures less than 1100 kN m^{-2}. With increase in pressure, it is observed that the correlations of Tandon *et al.* [1] begin to overpredict and the degree of deviation increases with increasing pressure. It is evident that the effects of pressure had not been adequately accounted for.

It may be readily shown that by taking $k = 3.0$, equation (4), which is of a much simpler form, represents the experimental data just as well as, if not better than, the predictions of Tandon *et al.* [1], but, as in the case with the predictions of Tandon *et al.* [1], there is an obvious pressure effect.

If, however, the value of k was allowed to vary with pressure, the resultant predictions become almost exact except for two situations which will be discussed. The predictions of this work are shown in Figs. 1 and 2, with the values of k as shown in Fig. 3. It should be pointed out that in the range of pressures 440–1210 kN m^{-2}, the value of k as given by Fig. 3 had a range of 2.6 to 3.0 in good agreement with $k = 2.5$ used in the case of the geothermal steam–water system [9]. As the detail experimental data were not available to the writer, Figs. 1 and 2 were reproduced from the article by Tandon *et al.* [1]. For the same reason, percentage deviations had not been evaluated. Nonetheless, inspection of these figures shows conclusively that equation (4) is superior.

The only two areas where there is no exact match between the experimental data and the predictions of equation (4) are in Fig. 1 and in the range where $x > 0.1$ in Fig. 2b for the case of 4700 kN m^{-2}. In Fig. 1, although the data were higher than predicted, the predictions of equation (4) are nevertheless far superior to all the others except the Smith correlation [18]. The predictions are of acceptable accuracy and are more accurate than those of Tandon *et al.* [1]. In Fig. 2b, in the case of $x > 0.1$ and 4700 kN m^{-2} pressure, there is observed some deviations between the Rouhani data and the predictions of this work. While it is not known exactly why this is so, these same data also showed inconsistency when compared with the correlations of Tandon *et al.* [1]. As discussed previously, Rouhani's data exhibit a progressive increase in deviations, with increasing pressure, from the correlation of Tandon *et al.* [1] in the pressure range of 3000–6000 kN m^{-2}. However, for the data in question, the expected trend was not followed.

CONCLUSIONS

In conclusion, equation (2) which was derived from a consideration of the ideal annular flow pattern may be modified to equation (4) with the incorporation of an empirical factor k which was allowed to vary with the system pressure. The resultant equation provides predictions which agree excellently with experimental data for the steam–water system in the pressure range 100–6000 kN m^{-2}. Equation (4) had previously been shown to represent data from a wide range of conditions including the Lockhart–Martinelli curve, when $k = 3.5$ [9, 19], two-phase flow of air and non-Newtonian fluids when k was taken to be dependent on X [20]. Moreover, equation (1), which is a different form of equation (4) had been shown also to be applicable to many gas–liquid flow situations including inclined upward and downward flows [10, 11, 23]. Thus equation (4) is not only a very simple form of equation, but also represents the results excellently and provides a link for the various void fraction results reported. While this and other work by the author [9–11, 19, 20] have provided the values of k for particular situations, further work is in progress in determining a more general and systematic method of predicting the value of k.

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